Differentially Rotating Plasma Rings with High Magnetic Energy Densities

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PLASMA DISKS AND RINGS WITH "HIGH" MAGNETIC ENERGY DENSITIES

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ABSTRACT

The nonlinear theory of rotating axisymmetric thin structures in which the magnetic field energy density i
comparable with the thermal plasma energy density is formulated. The only flow velocity included in the theory is th
velocity of rotation around a central object whose gravity is dominant. The periodic sequence, in the radial direction
of pairs of opposite current channels that can form is shown to lead to relatively large plasma density and pressur

Summary

Analytical (approximate) MHD time-stationary solutions have been found in the presence of a strongly gravitating central object exhibiting the following properties:

- "Crystal" Magnetic structures can form even when

$$p > \frac{B^2}{8\pi}$$

- "Corrugated" thin structures can form when

$$\frac{B_{\text{int}}^2}{8\pi} \sim \frac{B_{\text{ext}}^2}{8\pi} \sim p$$

- "Ring" structures can form when

$$\frac{B_{\text{int}}^2}{8\pi} > \frac{B_{\text{ext}}^2}{8\pi} \sim p$$

- We consider only a toroidal velocity to be present.
- Ideal MHD Equations (infinite conductivity) such that

$$\epsilon_m \equiv \frac{D_m}{v_A H}$$

is a small dimensionless parameter, D_m is the magnetic diffusion coefficient, v_A is the Alfvén velocity and H is the height of a "gaseous" disk.
- Ferraro's corotation theorem (infinite conductivity) yields

$$\Omega = \Omega(\Psi) = \Omega(\Psi_0 + \Psi_1) \simeq \Omega(\Psi_0) + \frac{d\Omega}{d\Psi_0}\Psi_1$$

where $\Omega(\Psi_0) = \Omega_k$.
- External field is current and force free, so that $\Psi_0 \simeq \Psi_0(R)$.

Basic Equations (cont.)

- Radial "Force Balance" (time-stationary) gives

$$2\Omega_k R_0(\delta\Omega)\rho \simeq \frac{\partial p}{\partial R}+\frac{1}{4\pi R_0^2}\left(\frac{\partial^2\Psi_1}{\partial R^2}+\frac{\partial^2\Psi_1}{\partial z^2}\right)\left(\frac{\partial\Psi_0}{\partial R}+\frac{\partial\Psi_1}{\partial R}\right)$$

- Vertical "Force Balance" gives

$$\frac{\partial p}{\partial z}+\Omega_k^2 zp+\frac{1}{4\pi R_0^2}\left(\frac{\partial^2\Psi_1}{\partial R^2}+\frac{\partial^2\Psi_1}{\partial z^2}\right)\frac{\partial\Psi_1}{\partial z}=0$$

- The internal/external division is then imposed on the density and the pressure.
- We consider $R_0 \gg 2H$, and perform a "multi-scale" analysis in the radial direction.

Dimensionless Variables and Relevant Scalings

- Length Scales:

$$\bar{R} = k_0(R - R_0) \qquad \text{and} \qquad \bar{z} = \frac{z}{\Delta_z}$$

where

$$k_0^2 \equiv \frac{3\Omega_k^2}{v_{A0}^2} \gg \frac{1}{R_0^2}$$

and

$$\Delta_z^2 \equiv \epsilon_z H_0^2$$

where

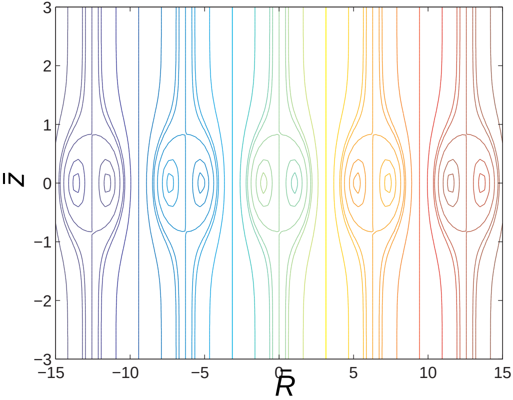
$$\epsilon_z^2 \equiv \frac{1}{3\beta_0} \qquad \text{and} \qquad H_0^2 \equiv 2\frac{p_0}{\rho_0}\frac{1}{\Omega_k^2}$$

so that

$$\Delta_z^2 \propto \frac{v_{A0}^2}{\Omega_k^2}$$

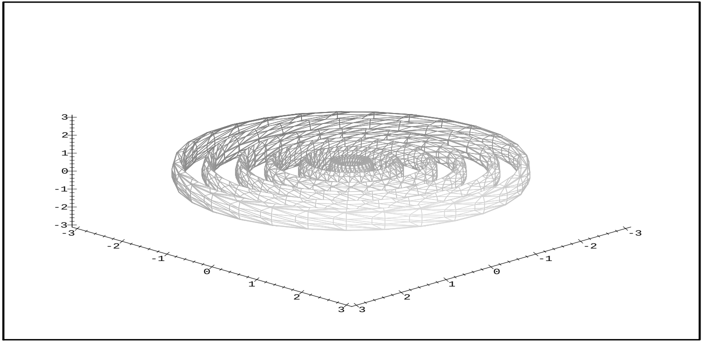
Crystal Structures

For $\beta \gg 1$ the vertical confinement of the plasma pressure is dominated by the gravitational term, recover "Gaseous Disk" with crystal magnetic structure.



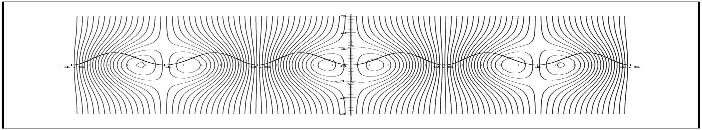
Ring structures

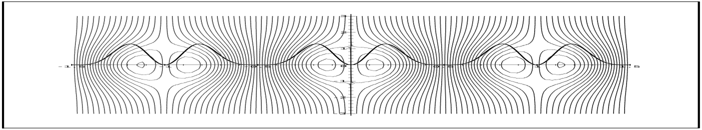
- When $B_{\rm int}^2/(8\pi) > B_{\rm ext}^2/(8\pi) \sim p$ then the vertical confinement of the plasma pressure is dominated by the Magnetic term.
- The vertical and radial scale-lengths are modified to be the Alfvén velocity formed from the geometric mean of the internal and external magnetic field.

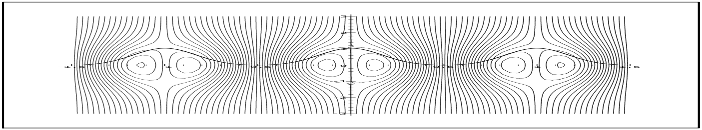


Density, Pressure, Temperature

There are many simple anaytical solutions with various radial profiles of the density, temperature, and pressure.

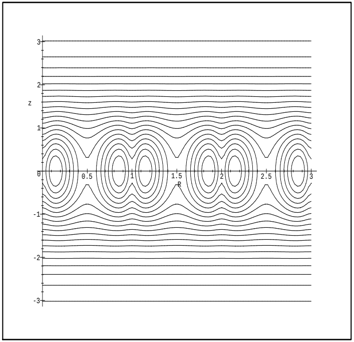


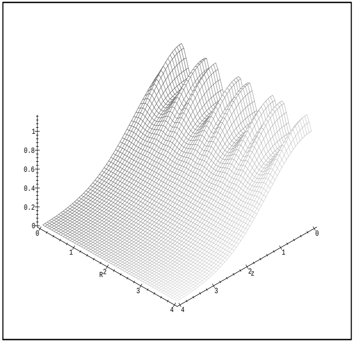




Corrugated Structures-Intermediate case

When $\beta \sim 1$ the case is intermediate, *i.e.* modulated density, temperature and pressure, but density does not go to zero.





Summary

- "Crystal" Magnetic structures can form even when $p > \frac{B^2}{8\pi}$
- "Corrugated" thin structures can form when $\frac{B^2_{\rm int}}{8\pi} \sim \frac{B^2_{\rm ext}}{8\pi} \sim p$
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- Interesting Questions
  - Self-gravitating disks (Bertin)
  - Can plasma collective modes arise from these configurations that can produce sufficient angular momentum transport?
  - What are the consequences on these structures of considering dusty plasmas?

For Further Reading I

- B. Coppi.
  “Crystal” magnetic structure in axisymmetric plasma accretion disks
  Physics of Plasmas. **12**, 057302 (2005).
- B. Coppi and F. Rousseau
  Plasma Disks and Rings with “High” Magnetic Energy Densities
  The Astrophysical Journal. **641**, 458 (2006).