Differentially Rotating Plasma Rings with High Magnetic Energy Densities

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Workshop on Magnetized Accretion Disks
PLASMA DISKS AND RINGS WITH “HIGH” MAGNETIC ENERGY DENSITIES

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ABSTRACT

The nonlinear theory of rotating axisymmetric thin structures in which the magnetic field energy density is comparable with the thermal plasma energy density is formulated. The only flow velocity included in the theory is the velocity of rotation around a central object whose gravity is dominant. The periodic sequence, in the radial direction, of pairs of opposite current channels that can form is shown to lead to relatively large plasma density and pressure.
Analytical (approximate) MHD time-stationary solutions have been found in the presence of a strongly gravitating central object exhibiting the following properties:

- “Crystal” Magnetic structures can form even when
  \[ p > \frac{B^2}{8\pi} \]

- “Corrugated” thin structures can form when
  \[ \frac{B_{\text{int}}^2}{8\pi} \sim \frac{B_{\text{ext}}^2}{8\pi} \sim p \]

- “Ring” structures can form when
  \[ \frac{B_{\text{int}}^2}{8\pi} > \frac{B_{\text{ext}}^2}{8\pi} \sim p \]
We consider only a toroidal velocity to be present.

Ideal MHD Equations (infinite conductivity) such that

\[ \epsilon_m \equiv \frac{D_m}{v_A H} \]

is a small dimensionless parameter, \( D_m \) is the magnetic diffusion coefficient, \( v_A \) is the Alfvén velocity and \( H \) is the height of a “gaseous” disk.

Ferraro’s corotation theorem (infinite conductivity) yields

\[ \Omega = \Omega(\psi) = \Omega(\psi_0 + \psi_1) \approx \Omega(\psi_0) + \frac{d\Omega}{d\psi_0} \psi_1 \]

where \( \Omega(\psi_0) = \Omega_k \).

External field is current and force free, so that \( \psi_0 \approx \psi_0(R) \).
Radial “Force Balance” (time-stationary) gives

\[ 2\Omega_k R_0 (\delta \Omega) \rho \approx \frac{\partial p}{\partial R} + \frac{1}{4\pi R_0^2} \left( \frac{\partial^2 \psi_1}{\partial R^2} + \frac{\partial^2 \psi_1}{\partial z^2} \right) \left( \frac{\partial \psi_0}{\partial R} + \frac{\partial \psi_1}{\partial R} \right) \]

Vertical “Force Balance” gives

\[ \frac{\partial p}{\partial z} + \Omega_k^2 z p + \frac{1}{4\pi R_0^2} \left( \frac{\partial^2 \psi_1}{\partial R^2} + \frac{\partial^2 \psi_1}{\partial z^2} \right) \frac{\partial \psi_1}{\partial z} = 0 \]

The internal/external division is then imposed on the density and the pressure.

We consider \( R_0 \gg 2H \), and perform a “multi-scale” analysis in the radial direction.
Dimensionless Variables and Relevant Scalings

- **Length Scales:**

\[ \bar{R} = k_0 (R - R_0) \quad \text{and} \quad \bar{z} = \frac{z}{\Delta z} \]

where

\[ k_0^2 \equiv \frac{3 \Omega_k^2}{\nu_{A0}^2} \gg \frac{1}{R_0^2} \]

and

\[ \Delta_z^2 \equiv \epsilon_z H_0^2 \]

where

\[ \epsilon_z^2 \equiv \frac{1}{3 \beta_0} \quad \text{and} \quad H_0^2 \equiv 2 \frac{\rho_0}{\rho_0} \frac{1}{\Omega_k^2} \]

so that

\[ \Delta_z^2 \propto \frac{\nu_{A0}^2}{\Omega_k^2} \]
For $\beta \gg 1$ the vertical confinement of the plasma pressure is dominated by the gravitational term, recover “Gaseous Disk” with crystal magnetic structure.
Ring structures

- When $B_{\text{int}}^2/(8\pi) > B_{\text{ext}}^2/(8\pi) \sim p$ then the vertical confinement of the plasma pressure is dominated by the Magnetic term.

- The vertical and radial scale-lengths are modified to be the Alfvén velocity formed from the geometric mean of the internal and external magnetic field.
There are many simple analytical solutions with various radial profiles of the density, temperature, and pressure.
When $\beta \sim 1$ the case is intermediate, \textit{i.e.} modulated density, temperature and pressure, but density does not go to zero.
“Crystal” Magnetic structures can form even when \( p > \frac{B^2}{8\pi} \)

“Corrugated” thin structures can form when \( \frac{B^2_{\text{int}}}{8\pi} \sim \frac{B^2_{\text{ext}}}{8\pi} \sim p \)

“Ring” structures can form when \( \frac{B^2_{\text{int}}}{8\pi} > \frac{B^2_{\text{ext}}}{8\pi} \sim p \)

Interesting Questions

- Self-gravitating disks (Bertin)
- Can plasma collective modes arise from these configurations that can produce sufficient angular momentum transport?
- What are the consequences on these structures of considering dusty plasmas?
For Further Reading I
